IERG4300
Web-Scale Information Analytics

Frequent Itemsets and Association Rule Mining

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Acknowledgements

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  - CS246 Mining Massive Data-sets, by Jure Leskovec, Stanford University.

with the author's permission.

Association Rule Discovery

Supermarket shelf management – Market-basket model:

- **Goal:** Identify items that are bought together by sufficiently many customers
- **Approach:** Process the sales data collected with barcode scanners to find dependencies among items
- **A classic rule:**
  - If one buys diaper and milk, then he is likely to buy beer
  - Don't be surprised if you find six-packs next to diapers!

TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Rules Discovered:
{Milk} --> {Coke}
{Diaper, Milk} --> {Beer}

The Market-Basket Model

- A large set of ***items***
  - e.g., things sold in a supermarket
- A large set of ***baskets***, each is a small subset of items
  - e.g., the things one customer buys on one day

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

- **A general many-many mapping (association) between two kinds of things**
  - But we ask about connections among “items”, not “baskets”

Association Rules: Approach

- **Given a set of baskets**
- Want to discover **association rules**
  - People who bought {x,y,z} tend to buy {v,w}
    - Amazon!
- **2-step approach:**
  - **1)** Find frequent **itemsets**
  - **2)** Generate **association rules**

TID	*Items*
1	Bread, Coke, Milk
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Rules Discovered:

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Applications – (1)

- **Items** = products; **Baskets** = sets of products someone bought in one trip to the store
- **Real market baskets:** Chain stores keep TBs of data about what customers buy together
  - Tells how typical customers navigate stores, lets them position tempting items
  - Suggests tie-in “tricks”, e.g., run sale on diapers and raise the price of beer
  - High **support** needed, or no $$’s
- **Amazon’s people who bought *X* also bought *Y***

Applications – (2)

- **Baskets** = sentences; **Items** = documents containing those sentences
  - Items that appear together too often could represent plagiarism
  - Notice items do not have to be “in” baskets
- **Baskets** = patients; **Items** = drugs & side-effects
  - Has been used to detect combinations of drugs that result in particular side-effects
  - **But requires extension:** Absence of an item needs to be observed as well as presence

Outline

First: Define

- Frequent itemsets
- Association rules:
  - Confidence, Support, Interestingness

Then: Algorithms for finding frequent itemsets

- Finding frequent pairs
- Apriori algorithm
- PCY algorithm + refinements

Frequent Itemsets

- **Simplest question:** Find sets of items that appear together "frequently" in baskets
- ***Support*** for itemset I: Number of baskets containing all items in I
  - Often expressed as a fraction of the total number of baskets
- Given a ***support threshold*** s, then sets of items that appear in at least s baskets are called ***frequent itemsets***

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

Example: Frequent Itemsets

- **Items** = {milk, coke, pepsi, beer, juice}
- **Minimum support** = 3 baskets

B_1 = {m, c, b}	B_2 = {m, p, j}
B_3 = {m, b}	B_4= {c, j}
B_5 = {m, p, b}	B_6 = {m, c, b, j}
B_7 = {c, b, j}	B_8 = {b, c}

- **Frequent itemsets:** {m}, {c}, {b}, {j}, {m,b} , {b,c} , {c,j}.

Association Rules

- **Association Rules:**
  If-then rules about the contents of baskets
- $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ means: "if a basket contains all of $i_1, \ldots, i_k$ then it is ***likely*** to contain j"
- **In practice there are many rules, want to find significant/interesting ones!**
- ***Confidence*** of this association rule is the probability of j given $I = \{i_1, \ldots, i_k\}$

$$\mathrm{conf}(I \rightarrow j) = \frac{\mathrm{support}(I \cup j)}{\mathrm{support}(I)} = \Pr[j \mid I]$$

*Note: $\mathrm{support}(I \cup j) =$ # (or %) of baskets contain BOTH I AND j

Interesting Association Rules

- **Not all high-confidence rules are interesting**
  - The rule $X \rightarrow milk$ may have high confidence for many itemsets X, because milk is just purchased very often (independent of X) and the confidence will be high
- **Interest** of an association rule $I \rightarrow j$:
  difference between its confidence and the fraction of baskets that contain j
  - Interesting rules are those with
    high positive or negative interest values

$$
\begin{aligned}
\text{Interest}(I \rightarrow j) &= \text{conf}(I \rightarrow j) - \Pr[j] \\
&= \Pr[j \mid I] - \Pr[j]
\end{aligned}
$$

Example: Confidence and Interest

$B_1 = \{m, c, b\}$ $B_2 = \{m, p, j\}$
$B_3 = \{m, b\}$ $B_4 = \{c, j\}$
$B_5 = \{m, p, b\}$ $B_6 = \{m, c, b, j\}$
$B_7 = \{c, b, j\}$ $B_8 = \{b, c\}$

- **Association rule: $\{m, b\} \rightarrow c$**
  - **Confidence =** 2/4 = 0.5
  - **Interest =** |0.5 – 5/8| = 1/8
    - Item *c* appears in 5/8 of the baskets
    - Rule is not very interesting!

Finding Association Rules

- **Problem: Find all association rules with support $\geq s$ and confidence $\geq c$**
  - **Note:** Support of an association rule is the support of the set of items on the left side
- **Hard part: Finding the frequent itemsets!**
  - If $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ has high support and confidence, then both $\{i_1, i_2, \ldots, i_k\}$ and $\{i_1, i_2, \ldots, i_k, j\}$ will be "frequent"

$$\text{conf}(I \rightarrow j) = \frac{\text{support}(I \cup j)}{\text{support}(I)}$$

Mining Association Rules

- **Step 1:** Find all frequent itemsets I
  - (we will explain this next)
- **Step 2: Rule generation**
  - For every subset A of I, generate a rule $A \rightarrow I \setminus A$
    - Since I is frequent, A is also frequent
    - **Variant 1:** Single pass to compute the rule confidence
      - conf(A,B→C,D) = supp(A,B,C,D)/supp(A,B)
    - **Variant 2:**
      - **Observation**:** If A,B,C→D is below confidence, so is A,B→C,D
      - Can generate "bigger" rules from smaller ones!
  - **Output the rules above the confidence threshold**

Mining Association Rules (cont'd)

- **Claim:** If A,B,C$\rightarrow$D is below confidence, so is A,B$\rightarrow$C,D

  Why ?

  Since Supp(ABC) =< Supp(AB)

Therefore:

  Conf.(ABC->D) = Supp(ABCD)/ Supp(ABC) >= Supp(ABCD)/Supp(AB) = Conf(AB->CD)

Thus,

  IF Conf(AB->CD) >= Threshold THEN Conf(ABC->D) also >= threshold ;

Equivalently,

  IF Conf(ABC->D) < Threshold then Conf(AB->CD) is also below threshold

This means we can first check Conf(AB->CD) if it is above threshold, we can simply generate additional rules, e.g. ABC->D, ABD->C.

=> Can generate "bigger" rules from smaller ones!

Example

$B_1 = \{m, a, b\}$ $B_2 = \{m, p, j\}$

$B_3 = \{m, a, b, n\}$ $B_4 = \{a, j\}$

$B_5 = \{m, p, b\}$ $B_6 = \{m, a, b, j\}$

$B_7 = \{a, b, j\}$ $B_8 = \{b, a\}$

- Min support $s=3$, confidence $c=0.75$
- **1) Frequent itemsets:**
  - {b,m} {a,b} {a,m} {a,j} {m,a,b}
- **2) Generate rules:**
  - $\cancel{b \rightarrow m: c=4/6}$ $b \rightarrow a: c=5/6$ $\cancel{b,a \rightarrow m: c=3/5}$
  - $m \rightarrow b: c=4/5$ … $b,m \rightarrow a: c=3/4$
  - $\cancel{b \rightarrow a,m: c=3/6}$

A Compact Way to store/track Frequent Itemsets

You only need to store the so-called:

***Maximal Frequent itemsets*:**

Definition: a Frequent set for which NO immediate superset is frequent

Nice Properties:

All subsets of a *Maximal Frequent itemset* are frequent

AND

Every *Frequent itemset* must be a subset of some *Maximal Frequent itemset*

=> By enumerating ALL subsets of all Maximal Frequent Itemsets, you will NOT miss any Frequent Itemset ! Also, every subset you got is a Frequent Itemset !

Example: Maximal Frequent Itemset

	Count	Maximal (s=3)
A	4	No
B	5	No
C	3	No
AB	4	Yes
AC	2	No
BC	3	Yes
ABC	2	No

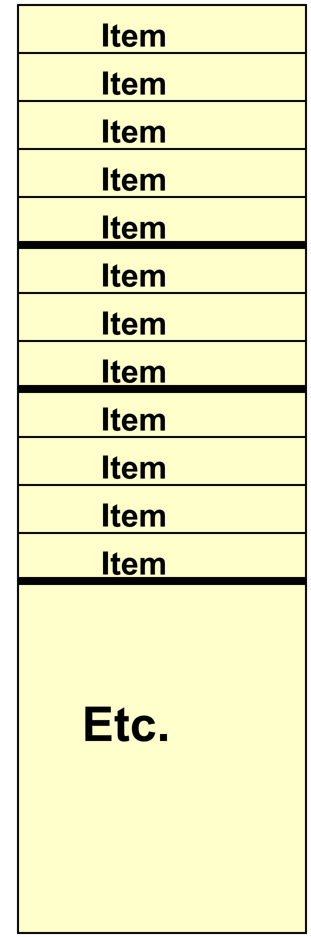
Frequent, but superset BC also frequent.

Frequent, and its only superset, ABC, not freq.

Finding Frequent Itemsets

Itemsets: Computation Model

- Back to finding frequent itemsets
- Typically, data is kept in flat files rather than in a database system:
  - Stored on disk
  - Stored basket-by-basket
  - Baskets are **small** but we have many baskets and many items
    - Expand baskets into pairs, triples, etc. as you read baskets
    - Use ***k*** nested loops to generate all sets of size ***k***



Note: We want to find frequent itemsets. To find them, we have to count them. To count them, we have to generate them.

Items are positive integers, and boundaries between baskets are –1.

Computation Model

- The true cost of mining disk-resident data is usually the number of disk I/O's
- In practice, association-rule algorithms read the data in *passes* – all baskets read in turn
- We measure the cost by the **number of passes** an algorithm makes over the data

Main-Memory Bottleneck

- For many frequent-itemset algorithms, **main-memory** is the critical resource
  - As we read baskets, we need to count something, e.g., occurrences of pairs of items
  - The number of different things we can count is limited by main memory
  - Swapping counts in/out is a disaster (why?)

Finding Frequent Pairs

- **The hardest problem often turns out to be finding the frequent pairs of items** $\{i_1, i_2\}$
  - **Why?** Often frequent pairs are common, frequent triples are rare
    - **Why?** Probability of being frequent drops exponentially with size; number of sets grows more slowly with size.
- Let's first concentrate on pairs, then extend to larger sets
- **The approach:**
  - We always need to generate all the itemsets
  - But we would only like to count/keep track of those itemsets that in the end turn out to be frequent

Naïve Algorithm

- **Naïve approach to finding frequent pairs**
- Read file once, counting in main memory the occurrences of each pair:
  - From each basket of *n* items, generate its *n(n-1)/2* pairs by two nested loops
- **Fails if (#items)^2 exceeds main memory**
  - **Remember:** #items can be 100K (Wal-Mart) or 10B (Web pages)
    - Suppose 10^5 items, counts are 4-byte integers
    - Number of pairs of items: $10^5(10^5-1)/2 = 5*10^9$
    - Therefore, $2*10^{10}$ (20 gigabytes) of memory needed

Counting Pairs in Memory

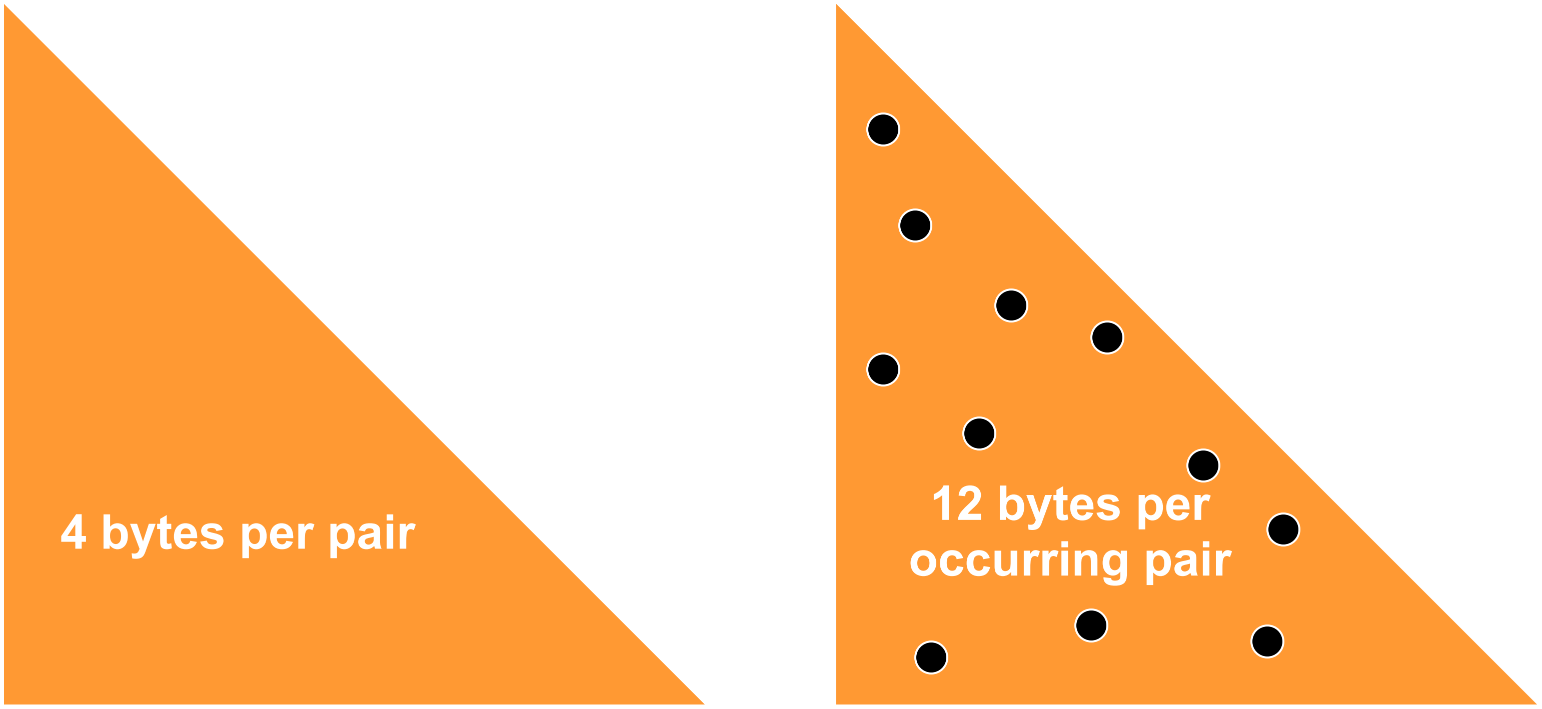
Two Approaches:

- **Approach 1:** Count all pairs using a matrix
- **Approach 2:** Keep a table of triples [i, j, c] = "the count of the pair of items {i, j} is c."
  - If integers and item ids are 4 bytes, we need approximately 12 bytes for pairs with count > 0
  - Plus some additional overhead for the hashtable

Note:

- **Approach 1** only requires 4 bytes per pair
- **Approach 2** uses 12 bytes per pair (but only for pairs with count > 0)

Comparing the 2 Approaches



Triangular Matrix Approach

Triangular Matrix Approach

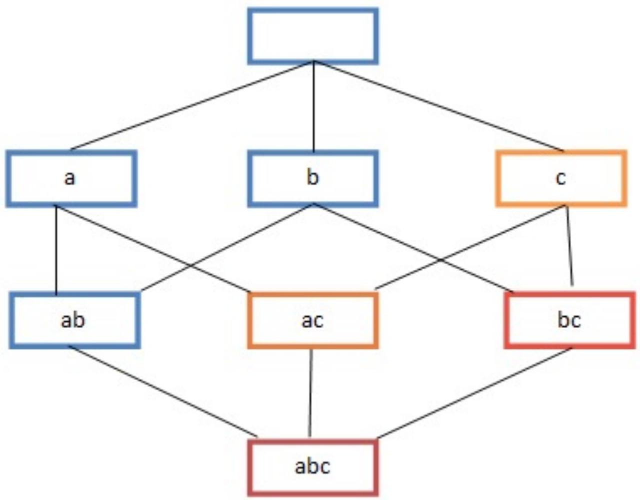
- n = total number items
- Count pair of items $\{i, j\}$ only if $i<j$

- Keep pair counts in lexicographic order:
  - $\{1,2\}, \{1,3\},\ldots, \{1,n\}, \{2,3\}, \{2,4\},\ldots,\{2,n\}, \{3,4\},\ldots$
- Pair $\{i, j\}$ is at position $(i-1)(n-i/2)+j-1$
- Total number of pairs $n(n-1)/2$; total bytes= $2n^2$
- **Triangular Matrix** requires 4 bytes per pair
- **Approach 2** uses 12 *bytes* per pair *(but only for pairs with count > 0)*
  - Beats triangular matrix if less than 1/3 of possible pairs actually occur

The A-Priori Algorithm

A-Priori Algorithm – (1)

- A **two-pass** approach called ***a-priori*** limits the need for main memory

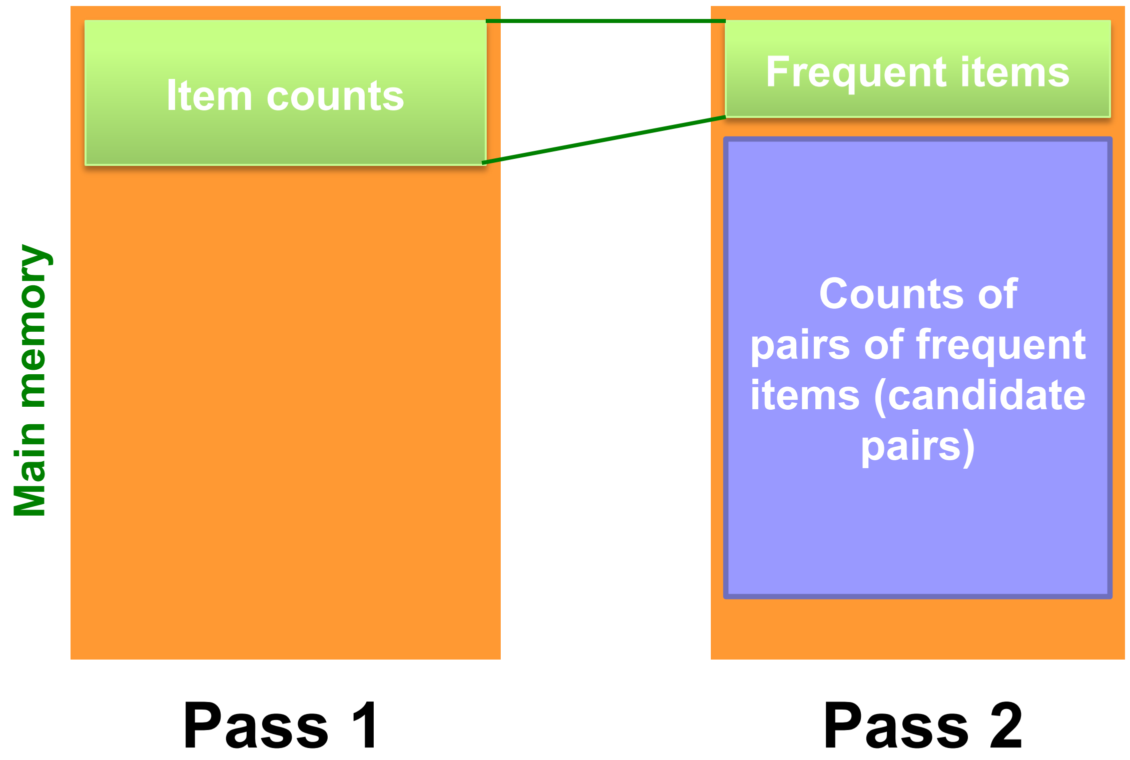


- **Key idea: *monotonicity***
  - If a set of items I appears at least s times, so does every **subset** J of I.
- **Contrapositive for pairs:**
  If item i does not appear in s baskets, then no pair including i can appear in s baskets

A-Priori Algorithm – (2)

- **Pass 1:** Read baskets and count in main memory the occurrences of each individual item
  - Requires only memory proportional to #items, usually enough memory even for 1 billon (= n) different types of items !
- **Items that appear >= s times are the frequent items**
- **Pass 2:** Read baskets again and count in main memory only those pairs where both elements are frequent (from Pass 1)
  - Requires memory proportional to square of **frequent** items only (for counts)
  - Plus a list of the frequent items (so you know what must be counted)

Main-Memory: Picture of A-Priori

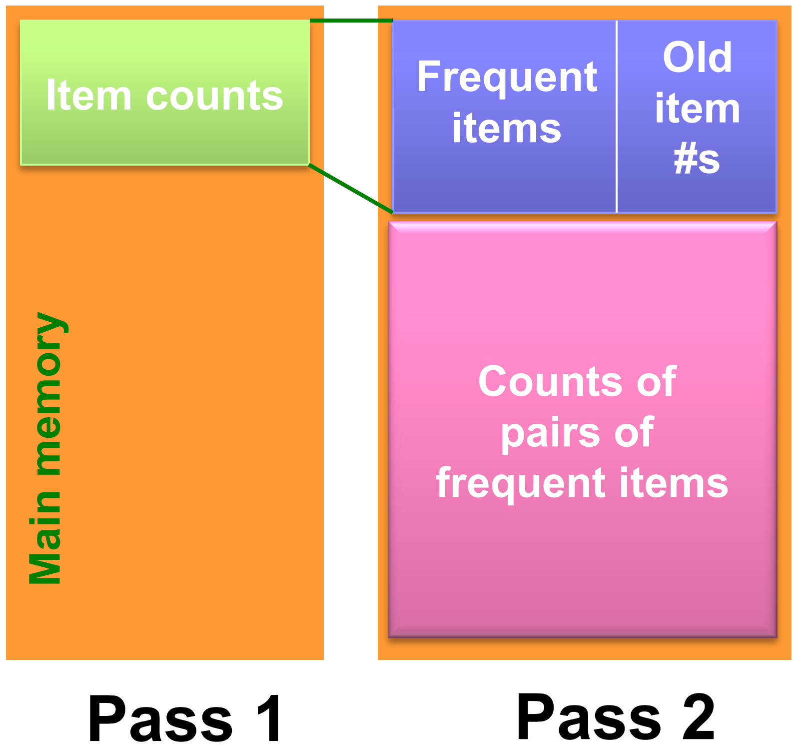


Detail for A-Priori

- You can use the triangular matrix method with:

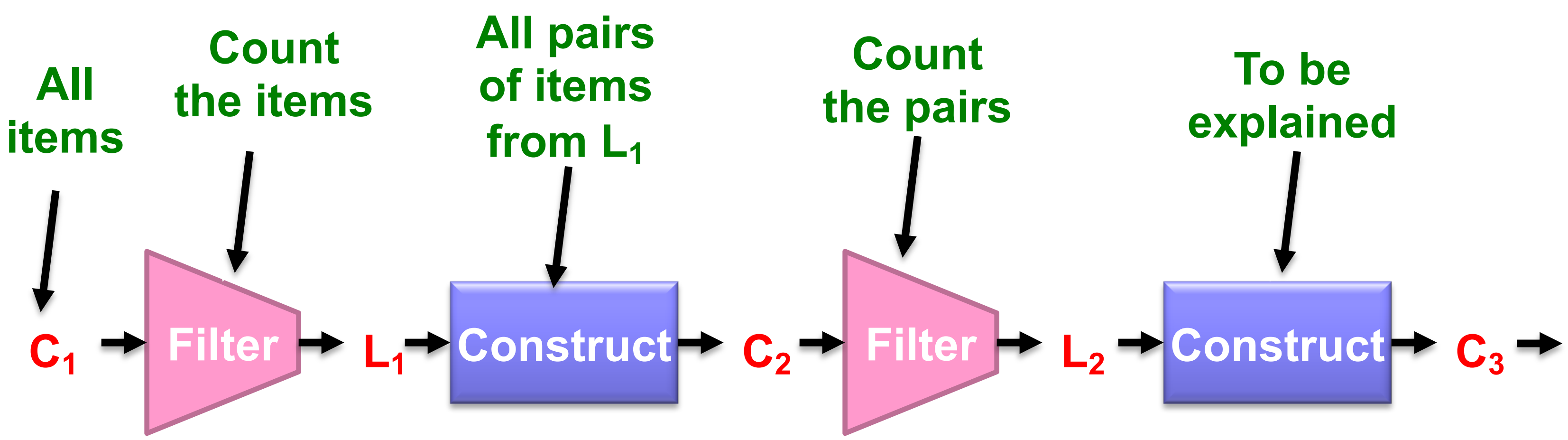
  n = number of frequent items

  - May save space compared with storing triples
- **Trick:** re-number frequent items 1,2,… and keep a table relating new numbers to original item numbers



Frequent Triples, Etc.

- **For each *k*, we construct two sets of *k-tuples*** (sets of size k):
  - C_k = ***candidate k-tuples*** = those that might be frequent sets (support $\geq$ s) based on information from the pass for k–1
  - L_k = the set of truly frequent k-tuples



Example

- **Hypothetical steps of the A-Priori algorithm**
  - C_1 = { {b} {c} {j} {m} {n} {p} }
  - Count the support of itemsets in C_1
  - Prune non-frequent: L_1 = { b, c, j, m }
  - Generate C_2 = { {b,c} {b,j} {b,m} {c,j} {c,m} {j,m} }
  - Count the support of itemsets in C_2
  - Prune non-frequent: L_2 = { {b,m} {b,c} {c,m} {c,j} }
  - Generate C_3 = { {b,c,m} <u>{b,c,j} {b,m,j} {c,m,j}</u> } **
  - Count the support of itemsets in C_3
  - Prune non-frequent: L_3 = { {b,c,m} }

** Note here we generate new candidates by generating C_k from L_{k-1} and L_1.
But that one can be more careful with candidate generation. For example, in C_3 we know {b,m,j} cannot be frequent since {m,j} is not frequent

A-Priori for All Frequent Itemsets

- One pass for each *k* (itemset size)
- Needs room in main memory to count each candidate *k*–tuple
- For typical market-basket data and reasonable support (e.g., 1%), $k = 2$ requires the most memory
- **Many possible extensions:**
  - Lower the support *s* as itemset gets bigger
  - Association rules with intervals:
    - For example: Men over 65 have 2 cars
  - Association rules when items are in a taxonomy
    - Bread, Butter → FruitJam
    - BakedGoods, MilkProduct → PreservedGoods

PCY (Park-Chen-Yu) Algorithm

PCY (Park-Chen-Yu) Algorithm

- **Observation:**
  In pass 1 of a-priori, most memory is idle
  - We store only individual item counts
  - **Can we use the idle memory to reduce memory required in pass 2?**
- **Pass 1 of PCY:** In addition to item counts, maintain a hash table with as many buckets as fit in memory
  - Keep a count for each bucket into which **pairs** of items are hashed
    - Just the count, not the pairs that hash to the bucket!

PCY Algorithm – First Pass

New in PCY (annotating the FOR (each pair of items) block)

```
FOR (each basket) :
    FOR (each item in the basket) :
        add 1 to item's count;
    FOR (each pair of items) :
        hash the pair to a bucket;
        add 1 to the count for that bucket;
```

- Pairs of items need to be generated from the input file; they are not present in the file
- We are not just interested in the presence of a pair, but we need to see whether it is present at least *s* (support) times

Observations about Buckets

- **If a bucket contains a frequent pair, then the bucket is surely frequent**
  - But we cannot based on the hash alone to eliminate any non-frequent member within this bucket
- Even without any frequent pair, a bucket can still be frequent (why?)
- **But, for a bucket with total count less than *s*, none of its pairs can be frequent**
  - Pairs that hash to a non-frequent bucket can be eliminated from the candidate list (even if the pair consists of 2 frequent items)
- **Pass 2:**
  Only count pairs that hash to frequent buckets

PCY Algorithm – Between Passes

- **Replace the buckets by a bit-vector:**
  - 1 means the bucket count exceeded the support s (a *frequent bucket*); 0 means it did not

- **4-byte integer counts are replaced by bits, so the bit-vector requires 1/32 of memory in Pass 1**

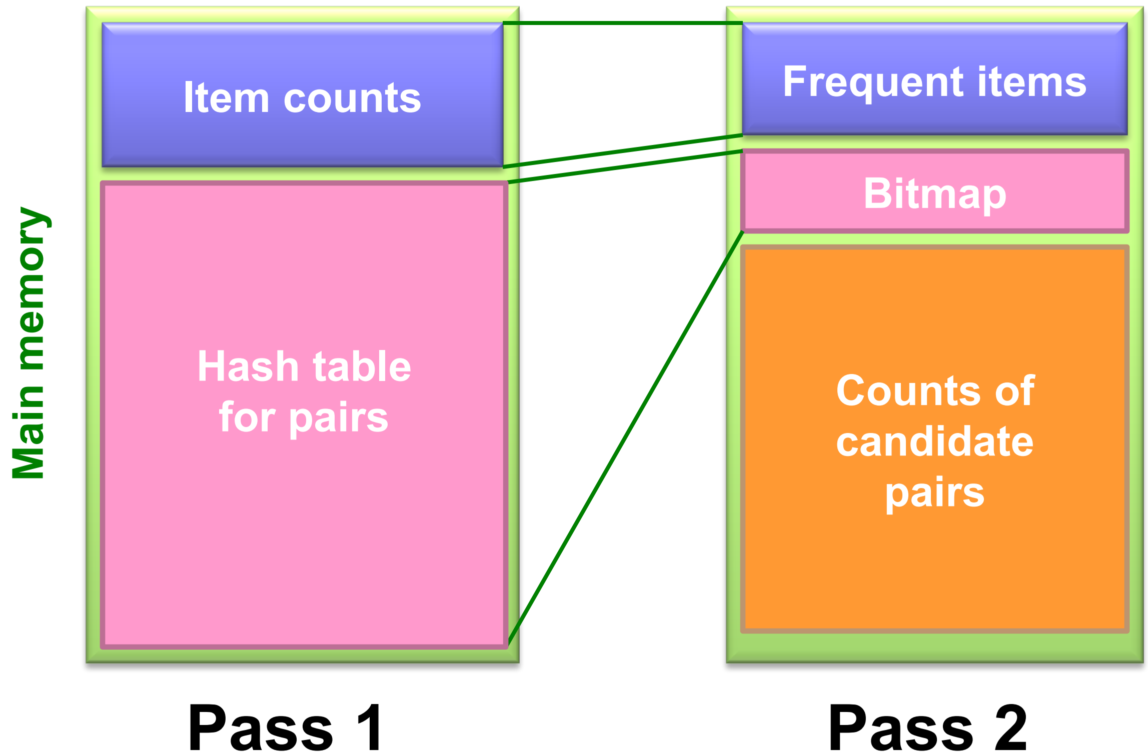
- Also, decide which items are frequent and list them for the second pass

PCY Algorithm – Pass 2

- Count all pairs $\{i, j\}$ that meet the conditions for being a **candidate pair**:
  1. Both i and j are frequent items
  2. The pair $\{i, j\}$ hashes to a bucket whose bit in the bit vector is 1 (i.e., frequent bucket)

- **Both conditions are necessary for the pair to have a chance of being frequent**

Main-Memory: Picture of PCY



Main-Memory Details

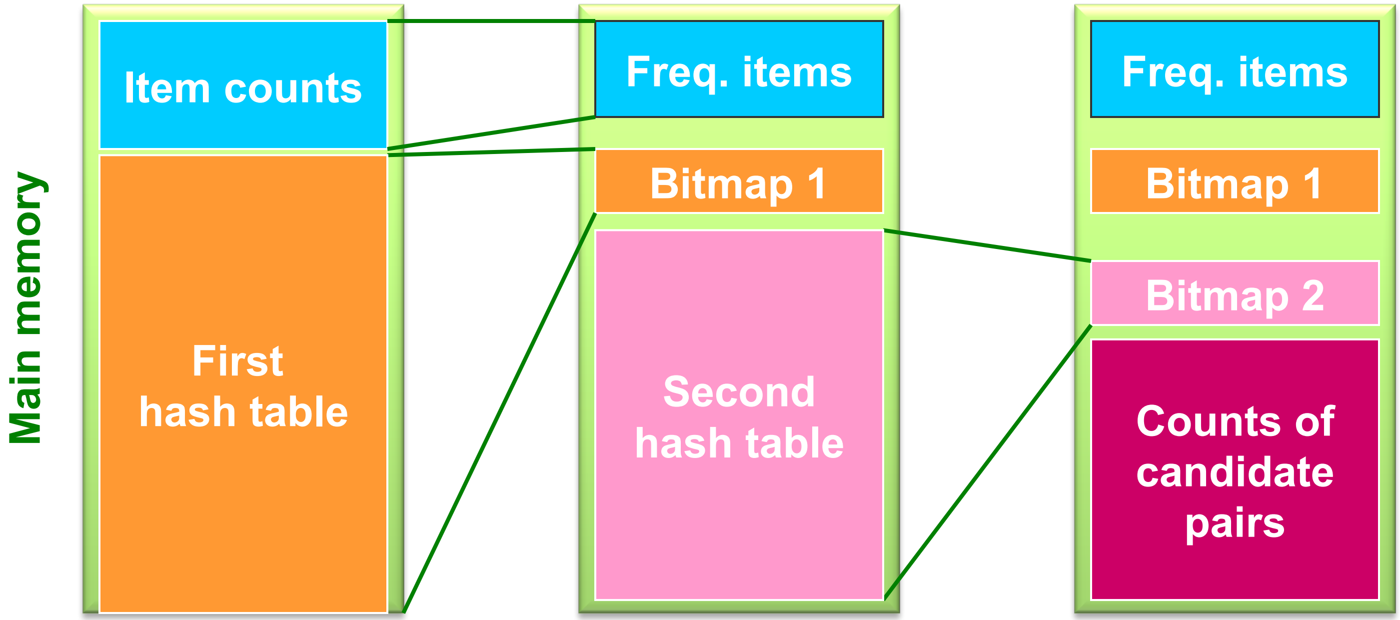
- **Buckets require a few bytes each:**
  - **Note:** we don't have to count past s
  - #buckets is O(main-memory size)

- On second pass, a table of (item, item, count) triples is essential (we cannot use triangular matrix approach, why?)
  - Thus, hash table must eliminate approx. 2/3 of the candidate pairs for PCY to beat A-Priori.

Refinement: Multistage Algorithm

- **Limit the number of candidates to be counted**
  - **Remember:** Memory is the bottleneck
  - Still need to generate all the itemsets but we only want to count/keep track of the ones that are frequent
- **Key idea:** After Pass 1 of PCY, rehash only those pairs that **qualify** for Pass 2 of PCY
  - *i* and *j* are frequent, and
  - *{i, j}* hashes to a frequent bucket from Pass 1
- On middle pass (i.e. the new Pass 2), fewer pairs contribute to buckets, so fewer ***false positives***
- **Requires 3 passes over the data**

Main-Memory: Multistage



Pass 1

Count items
Hash pairs {i,j}

Pass 2

Hash pairs {i,j}
into Hash2 iff:
i,j are frequent,
&& {i,j} hashes to
freq. bucket in B1

Pass 3

Count pairs {i,j} iff:
i,j are frequent,
&& {i,j} hashes to
freq. bucket in B1
&& {i,j} hashes to
freq. bucket in B2

Multistage – Pass 3

- **Count only those pairs $\{i, j\}$ that satisfy these candidate pair conditions:**
  1. Both i and j are frequent items
  2. Using the first hash function, the pair hashes to a bucket whose bit in the first bit-vector is 1.
  3. Using the second hash function, the pair hashes to a bucket whose bit in the second bit-vector is 1.

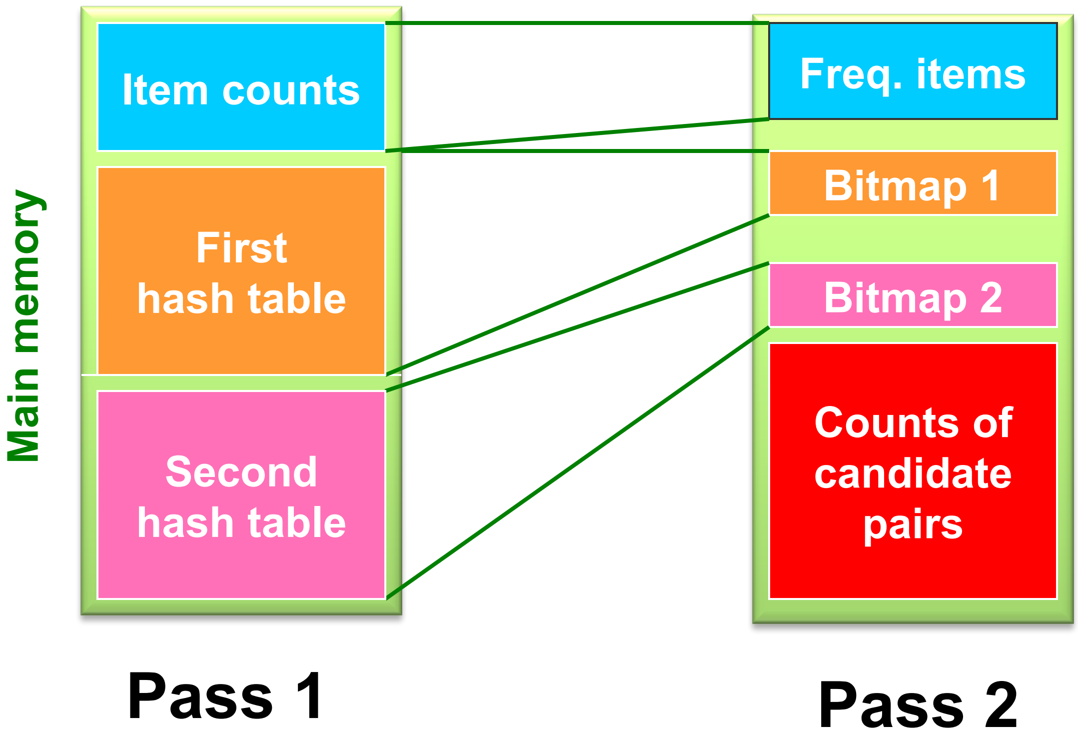
Important Points

1. **The two hash functions have to be independent**
2. **We need to check both hashes on the third pass**
   - If not, we would end up counting pairs of frequent items that hashed first to an infrequent bucket (during Pass 1) but happened to hash to a frequent bucket during the new Pass 2.

Refinement: Multihash

- **Key idea:** Use several independent hash tables on the first pass
- **Risk:** Halving the number of buckets doubles the average count
  - We have to be sure most buckets will still not reach count s
- If so, we can get a benefit like multistage,
  but in only 2 passes

Main-Memory: Multihash



PCY: Extensions

- Either **multistage** or **multihash** can use more than two hash functions
- In **multistage**, there is a point of diminishing returns, since the bit-vectors eventually consume all of main memory
- For **multihash**, the bit-vectors occupy exactly what one PCY bitmap does, but given the constant amount of main memory to hold all hash tables, too many hash functions makes all counts $\geq$ *s, and thus, fails to eliminate any non-frequent pairs !*

Frequent Itemsets in $\leq$ 2 Passes

Frequent Itemsets in $\leq$ 2 Passes

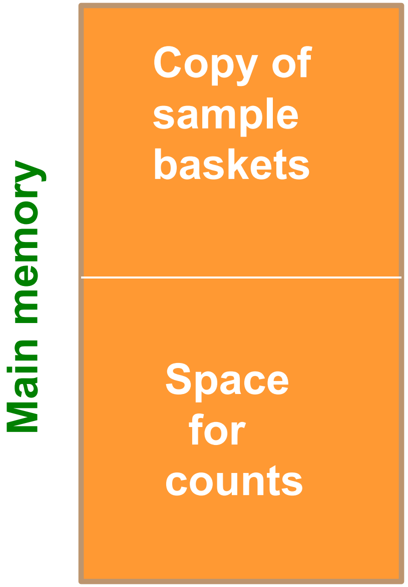
- A-Priori, PCY, etc., take *k* passes to find frequent itemsets of size *k*

- **Can we use fewer passes?**

- Use 2 or fewer passes for all sizes,
  but may miss some frequent itemsets
  - Random sampling
  - SON (Savasere, Omiecinski, and Navathe)
  - Toivonen (see textbook [MMDS Ch6])

Random Sampling (1)

- Take a random sample of the market baskets
- Run A-priori or one of its improvements in main memory
  - So we don't pay for disk I/O each time we increase the size of itemsets
  - MUST reduce support threshold proportionally to match the sample size



Random Sampling (2)

- Optionally, verify that the candidate pairs are truly frequent in the entire data set by a second pass (avoid false positives)

- But you don’t catch sets frequent in the whole but not in the sample
  - Smaller threshold, e.g., *s*/125, helps catch more truly frequent itemsets
    - But requires more space

SON Algorithm – (1)

- Repeatedly read small subsets of the baskets into main memory and run an in-memory algorithm to find all frequent itemsets
  - Note: we are not sampling, but processing the entire file in memory-sized chunks

- An itemset becomes a candidate if it is found to be frequent in *any* one or more subsets of the baskets.

SON Algorithm – (2)

- On a **second pass**, count all the candidate itemsets and determine which are frequent in the entire set
- **Key idea:** an itemset cannot be frequent in the entire set of baskets unless it is frequent in at least one subset.

SON – Distributed Version

- SON lends itself to distributed/ parallel implementation, e.g. using MapReduce

- Baskets distributed among many nodes
  - Compute frequent itemsets at each node
  - Distribute candidate itemsets to all nodes
  - Accumulate the counts of all candidates
- Can be done with two MapReduce jobs:
  - First MapReduce job to produce the candidate itemsets
  - Second MapReduce job to calculate the true frequent itemsets.

SON: Map/Reduce

- **Job 1:** Find candidate itemsets
  - Map?
  - Reduce?

- **Job 2:** Find true frequent itemsets
  - Map?
  - Reduce?

SON: MapReduce Implementation

Mapper for Job 1

- Run A-Priori algorithm on the chunk using support threshold ps
- Output the frequent itemsets for that chunk (F, c), where F is the key (itemset) and c is count (or proportion)

Reducer for Job 1

- Output the candidate itemsets to be verified in the Job 2
- Given (F,c), discard c and output all candidate itemsets F's

SON: MapReduce Implementation (cont'd)

Mapper for Job 2

- For all the candidate itemsets produced by Job 1, count the frequency in local chunk

Reducer for Job 2

- Aggregate the o/p of the Mapper of Job 2 and sum the count to get the frequency of each candidate itemsets across the entire input file
- Filter out the itemsets with support smaller than *s*